

# Canonical quantization of electromagnetic field in the presence of absorbing bi-anisotropic multilayer magnetodielectric media

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## Abstract

A bi-anisotropic magnetodielectric medium is modeled by two independent set of three dimensional harmonic oscillators .A fully canonical quantization of electromagnetic field is demonstrated in the presence of a bi-anisotropic magnetodielectric medium. The electric and magnetic polarization fields of the medium are obtained in terms of the dynamical variable modeling the medium. The Heisenberg equations of the system are solved for a multilayer bi-anisotropic magnetodielectric medium.

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## 1 Introduction

It is well known that the quantum properties of electromagnetic field can be influenced by the presence of magnetodielectric media. Typical examples are the casimir forces [1]-[4] and the modification of the spontaneous emission rate of excited atoms in the presence of polarizable and magnetizable media [5]-[16].

There are mainly three approaches to quantize electromagnetic field in the

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presence of magnetodielectric bodies. One approach is the damped polarization model [17]-[20]. In this method the electric polarization of the medium is represented by a quantum field and absorptivity character of the dielectric medium is described by interaction between the polarization field with a heat bath containing a continuous set of harmonic oscillators. In this approach a canonical quantization is formulated for electromagnetic field and the polarizable medium. The dielectric function of the medium is obtained in terms of the coupling function of the heat bath and the electric polarization field such that it satisfy the Kramers-Kronig relations.

In a second approach, to quantize electromagnetic field in the presence of magnetodielectric media, by adding the noise electric and magnetic polarization densities to the classical constitutive equations of the medium, these equations is taken as the definitions of electric and magnetic polarization operators [21]-[28]. The noise polarizations are related to two independent sets of bosonic operators. Then, combination of the Maxwell equations and the constitutive equations in frequency domain, gives the electromagnetic field operators in terms of the noise polarizations and the classical Green tensor. Suitable commutation relations are imposed on the bosonic operators such that the commutation relations between electromagnetic field operators become identical with those in free space.

In a third scheme a fully canonical quantization of electromagnetic field has been introduced in the presence of an anisotropic polarizable and magnetizable medium[29],[30]. In this method the medium is modeled by two independent reservoirs. Each reservoir contains a continuum of three dimensional harmonic oscillators. The two reservoirs describe polarizability and magnetizability of the medium. In contrast of the damped polarization model, introduced above, in this approach the electric and magnetic polarization fields of the medium do not need to appear explicitly in the Lagrangian of the total system as a part of the degrees of freedom of the medium. The contribution of the medium in the Lagrangian of the total system is related only to the two reservoirs and these reservoirs completely constitute the degrees of freedom of the medium.

In the present paper, we generalize the third approach [29]-[31] for bi-anisotropic magnetodielectric media and then, the Maxwell equations in the Heisenberg picture are solved exactly for a multilayer medium.

## 2 The Lagrangian of the system

In this section we generalize the canonical quantization method in the reference [29] for bi-anisotropic media. In this approach the magneto-dielectric is modeled by two continuum collection of three dimensional harmonic oscillators and the total lagrangian is proposed as

$$L(t) = \int d^3r [\mathcal{L}_s + \mathcal{L}_{em} + \mathcal{L}_{int}]. \quad (1)$$

The first term in the integrand is the contribution related to the medium and is written as

$$\mathcal{L}_s = \int_0^\infty d\omega \sum_{i=1}^2 \left[ \frac{1}{2} \dot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, t) \cdot \dot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, t) - \frac{1}{2} \omega^2 \mathbf{X}_\omega^{(i)}(\mathbf{r}, t) \cdot \mathbf{X}_\omega^{(i)}(\mathbf{r}, t) \right] \quad (2)$$

where  $\mathbf{X}_\omega^{(i)}(\mathbf{r}, t)$  for  $i = 1, 2$  is the dynamical variables of the harmonic oscillator labeled by the continuous parameter  $\omega$ . The second part in the integrand (1) is the Lagrangian density of the electromagnetic field which is written as usual way as

$$\mathcal{L}_{em} = \frac{1}{2} \varepsilon_0 \mathbf{E}^2 - \frac{\mathbf{B}^2}{2\mu_0}, \quad (3)$$

where  $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi$ ,  $\mathbf{B} = \nabla \times \mathbf{A}$  are electric and magnetic fields respectively and  $\mathbf{A}$  and  $\varphi$  are the vector and the scalar potentials. The last term  $\mathcal{L}_{int}$  in (1) is describing the interaction between the bi-anisotropic magnetodielectric medium and the electromagnetic field and is as follows

$$\begin{aligned} \mathcal{L}_{int} = & \int_0^\infty d\omega \sum_{i=1}^2 \left( \sum_{m,n=1}^3 f_i^{mn}(\omega, \mathbf{r}) E_m(\mathbf{r}, t) X_{\omega n}^{(i)}(\mathbf{r}, t) \right) \\ & + \int_0^\infty d\omega \sum_{i=1}^2 \left( \sum_{m,n=1}^3 g_i^{mn}(\omega, \mathbf{r}) B_m(\mathbf{r}, t) X_{\omega n}^{(i)}(\mathbf{r}, t) \right) \end{aligned} \quad (4)$$

where  $\mathbf{f}_i, \mathbf{g}_i, i = 1, 2$  are the coupling tensors of the second rank between the electromagnetic field and the medium. The interaction part (4) is the generalization of the Lagrangian that has previously been used for anisotropic spatially dispersion magneto-dielectric media[29].

As mentioned above, in contrast of the damped polarization method, in this scheme the electric and magnetic polarization fields are not appeared explicitly in the Lagrangian density of the total system and the contribution of the medium in the lagrangian merely is  $\mathcal{L}_s$  containing two independent reservoirs.

### 3 Classical Euler-Lagrange equations

Using the Lagrangian density (2)-(4) it can be shown that the classical Euler-Lagrange equations for the scalar and vector potentials  $\varphi$  and  $\mathbf{A}$  leads to the Gauss' law and the Maxwell equation, respectively as

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0 \quad (5)$$

$$\nabla \times \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \frac{\partial}{\partial t} (\varepsilon_0 \mathbf{E} + \mathbf{P}), \quad (6)$$

where  $\mathbf{P}$  and  $\mathbf{M}$  are the electric and magnetic polarization densities of the medium which are defined in the terms of the oscillators modeling the medium as

$$\begin{aligned} \mathbf{P}(\mathbf{r}, t) &= \int_0^\infty d\omega \sum_{i=1}^2 \mathbf{f}_i(\omega, \mathbf{r}) \cdot \mathbf{X}_\omega^{(i)}(\mathbf{r}, t) \\ \mathbf{M}(\mathbf{r}, t) &= \int_0^\infty d\omega \sum_{i=1}^2 \mathbf{g}_i(\omega, \mathbf{r}) \cdot \mathbf{X}_\omega^{(i)}(\mathbf{r}, t). \end{aligned} \quad (7)$$

Also the Euler-Lagrange equations for the dynamical variables  $\mathbf{X}_\omega^{(i)}$ ,  $i = 1, 2$  leads to

$$\ddot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, t) + \omega^2 \mathbf{X}_\omega^{(i)}(\mathbf{r}, t) = \mathbf{f}_i^t(\omega, \mathbf{r}) \cdot \mathbf{E}(\mathbf{r}, t) + \mathbf{g}_i^t(\omega, \mathbf{r}) \cdot \mathbf{B}(\mathbf{r}, t). \quad (8)$$

where the superscript  $t$  imply the transposition. The formally solution of the differential equations ( 8) can be written as

$$\begin{aligned} \mathbf{X}_\omega^{(i)}(\mathbf{r}, t) &= \mathbf{X}_\omega^{(i)}(\mathbf{r}, 0) \cos \omega t + \frac{\sin \omega t}{\omega} \dot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, 0) \\ &+ \int_0^t dt' \frac{\sin \omega(t-t')}{\omega} [\mathbf{f}_i^t(\omega, \mathbf{r}) \cdot \mathbf{E}(\mathbf{r}, t') + \mathbf{g}_i^t(\omega, \mathbf{r}) \cdot \mathbf{B}(\mathbf{r}, t')] \end{aligned} \quad (9)$$

If one substitute the solutions (9) into the definitions (7), the constitutive relations of the bi-anisotropic magneto-dielectric medium are obtained as

$$\begin{aligned}\mathbf{P}(\mathbf{r}, t) &= \mathbf{P}_N(\mathbf{r}, t) + \int_0^{|t|} dt' [\chi^{(1)}(\mathbf{r}, |t| - t') \cdot \mathbf{E}(\mathbf{r}, \pm t') + \chi^{(2)}(\mathbf{r}, |t| - t') \cdot \mathbf{B}(\mathbf{r}, \pm t')] \\ \mathbf{M}(\mathbf{r}, t) &= \mathbf{M}_N(\mathbf{r}, t) + \int_0^{|t|} dt' [\chi^{(3)}(\mathbf{r}, |t| - t') \cdot \mathbf{E}(\mathbf{r}, \pm t') + \chi^{(4)}(\mathbf{r}, |t| - t') \cdot \mathbf{B}(\mathbf{r}, \pm t')]\end{aligned}\tag{10}$$

where the upper (lower) sign is for  $t > 0$  ( $t < 0$ ) and the tensors  $\chi^{(i)}$ ,  $i = 1, 2, 3, 4$  are the susceptibility tensors of the medium and are defined in terms of the coupling tensors of the electromagnetic field and the medium as the following

$$\begin{aligned}\chi^{(1)}(\mathbf{r}, t) &= \int_0^\infty d\omega \frac{\sin \omega t}{\omega} [\mathbf{f}_1(\omega, \mathbf{r}) \cdot \mathbf{f}_1^t(\omega, \mathbf{r}) + \mathbf{f}_2(\omega, \mathbf{r}) \cdot \mathbf{f}_2^t(\omega, \mathbf{r})] \\ \chi^{(4)}(\mathbf{r}, t) &= \int_0^\infty d\omega \frac{\sin \omega t}{\omega} [\mathbf{g}_1(\omega, \mathbf{r}) \cdot \mathbf{g}_1^t(\omega, \mathbf{r}) + \mathbf{g}_2(\omega, \mathbf{r}) \cdot \mathbf{g}_2^t(\omega, \mathbf{r})] \\ \chi^{(2)}(\mathbf{r}, t) &= (\chi^{(3)})^t(\mathbf{r}, t) = \\ &= \int_0^\infty d\omega \frac{\sin \omega t}{\omega} [\mathbf{f}_1(\omega, \mathbf{r}) \cdot \mathbf{g}_1^t(\omega, \mathbf{r}) + \mathbf{f}_2(\omega, \mathbf{r}) \cdot \mathbf{g}_2^t(\omega, \mathbf{r})]\end{aligned}\tag{11}$$

In the constitutive relations (10) the fields  $\mathbf{P}_N$  and  $\mathbf{M}_N$  are the noise polarization densities which can be written as

$$\begin{aligned}\mathbf{P}_N(\mathbf{r}, t) &= \int_0^\infty d\omega \sum_{i=1}^2 \left[ \mathbf{f}_i(\omega, \mathbf{r}) \cdot \left( \mathbf{X}_\omega^{(i)}(\mathbf{r}, 0) \cos \omega t + \dot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, 0) \frac{\sin \omega t}{\omega} \right) \right] \\ \mathbf{M}_N(\mathbf{r}, t) &= \int_0^\infty d\omega \sum_{i=1}^2 \left[ \mathbf{g}_i(\omega, \mathbf{r}) \cdot \left( \mathbf{X}_\omega^{(i)}(\mathbf{r}, 0) \cos \omega t + \dot{\mathbf{X}}_\omega^{(i)}(\mathbf{r}, 0) \frac{\sin \omega t}{\omega} \right) \right]\end{aligned}\tag{12}$$

An anisotropic magnetodielectric medium is the medium that its constitutive relation are as

$$\begin{aligned}\mathbf{P}(\mathbf{r}, t) &= \mathbf{P}_N(\mathbf{r}, t) + \int_0^{|t|} dt' \chi_e(\mathbf{r}, |t| - t') \cdot \mathbf{E}(\mathbf{r}, \pm t') \\ \mathbf{M}(\mathbf{r}, t) &= \mathbf{M}_N(\mathbf{r}, t) + \int_0^{|t|} dt' \chi_m(\mathbf{r}, |t| - t') \mathbf{B}(\mathbf{r}, \pm t')\end{aligned}\tag{13}$$

where  $\chi_e$  and  $\chi_m$  are the electric and magnetic susceptibilities tensors. In order to obtain Eq.(13), in the Heisenberg picture, in the Lagrangian density one set of the oscillator modeling the medium should be coupled to the electric field and the other should be coupled to the magnetic field, separately [29]. A bi-anisotropic magnetodielectric medium is meant the medium that the constitutive relations are as (10), where  $\chi^{(i)}$   $i = 1, 2, 3, 4$  are tensors of the second rank. that is, in bi-anisotropic magnetodielectric medium the electric and magnetic polarization densities are dependent on both the electric and magnetic field. In order to obtain Eqs. (10), in the Heisenberg picture, both collections of oscillators modeling the medium should be coupled with both electric and magnetic field. The oscillators modeling the medium describe the absorption of the energy of the electromagnetic field by the medium due to temporal dispersive property. Also as it is clear from Eqs.(7) the electric and magnetic polarization densities are defined in terms of the oscillators modeling the medium. Therefore the two collections of the oscillators describe polarizability, magnetizability and the absorption due to temporal dispersive property of the magnetodielectric medium. In fact this scheme of quantization is a generalization of the Caldeira-Legget model for dissipative system [32] and [33]. In the Caldeira-Legget model the absorbing environment of a main dissipative system is modeled by a set of harmonic oscillators. The oscillators describe the absorption of the energy of the main system by the environment. In the present quantization our main dissipative system is the electromagnetic field and the magnetodielectric medium play the role of the absorbing environment. Accordingly the oscillators modeling the medium describe the absorption of the energy by the absorbing magnetodielectric medium.

## 4 Canonical Quantization

To have a canonical quantization of the combined system, that is the electromagnetic field and the medium, we are confronted with the usual problem that the scalar potential  $\varphi$  does not possess a canonically conjugate variable, since the lagrangian density of the total system does not contain the time derivative of the the scalar potential. To overcome this , as the usual way, the Gauss' law (5) together with the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$  can be used to eliminate the extra degree's of freedom from the Lagrangian of the total system. Applying the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$  in the Gauss' law (5) the scalar potential can be chosen as

$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\nabla' \cdot \mathbf{P}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} \quad (14)$$

substitution of the constraint (14) into the Lagrangian (1)-(4) and apply several integration by parts, for the total Lagrangian of the system, we have

$$\begin{aligned} L(t) = & \int d^3r \left[ \frac{1}{2}\epsilon_0 \left( \frac{\partial \mathbf{A}}{\partial t} \right)^2 - \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} - \frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{P}^T + (\nabla \times \mathbf{A}) \cdot \mathbf{M} \right] \\ & + \int_0^\infty d\omega \sum_{i=1}^2 \left[ \frac{1}{2} (\dot{\mathbf{X}}_\omega^{(i)})^2 - \frac{1}{2} \omega^2 (\mathbf{X}_\omega^{(i)})^2 \right] \\ & - \frac{1}{4\pi\epsilon_0} \int d^3r \int d^3r' \frac{\nabla \cdot \mathbf{P}(\mathbf{r}, t) \nabla' \cdot \mathbf{P}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} \end{aligned} \quad (15)$$

where the polarization fields  $\mathbf{P}, \mathbf{M}$  are given by (7) and  $\mathbf{P}^T$  is the transverse component of the electric polarization field  $\mathbf{P}$  defined by

$$P_i^T(\mathbf{r}, t) = \sum_{j=1}^3 \int d^3r' \delta_{ij}^T(\mathbf{r}, \mathbf{r}') P_j(\mathbf{r}', t) \quad (16)$$

where

$$\delta_{ij}^T(\mathbf{r}, \mathbf{r}') = \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') - \frac{1}{4\pi} \frac{\partial}{\partial x_i} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \frac{\partial}{\partial x_j'} \quad (17)$$

is the transverse Dirac operator. Now the Lagrangian (15) possess only the vector potential as the dynamical variable of the electromagnetic field. One

can compute the conjugate variables of the fields  $\mathbf{A}, \mathbf{X}^{(1)}$  and  $\mathbf{X}^{(2)}$  respectively as

$$-\mathbf{D} = \frac{\delta L}{\delta \dot{\mathbf{A}}} = \varepsilon_0 \dot{\mathbf{A}} - \mathbf{P}^T \quad (18)$$

$$\mathbf{Q}_\omega^{(i)} = \frac{\delta L}{\delta \dot{\mathbf{X}}_\omega^{(i)}} = \dot{\mathbf{X}}_\omega^{(i)} \quad i = 1, 2 \quad (19)$$

Having the conjugate variables of the combined system, as the standard fashion, we postulate the following canonical commutation relations on the cartesian components of the field operators of the system

$$[A_i(\mathbf{r}, t), -D_j(\mathbf{r}', t)] = i\hbar \delta_{ij}^T(\mathbf{r}, \mathbf{r}') \quad (20)$$

$$[\mathbf{X}_\omega^{(i)}(\mathbf{r}, t), \mathbf{Q}_{\omega'}^{(j)}(\mathbf{r}', t)] = i\hbar I \delta_{ij} \delta(\omega - \omega') \delta(\mathbf{r} - \mathbf{r}') \quad (21)$$

where  $I$  is the unit matrix. Using These commutation relations and the Hamiltonian

$$\begin{aligned} H(t) &= \int d^3r \left[ \int_0^\infty d\omega \sum_{i=1}^2 \mathbf{Q}_\omega^{(i)} \cdot \dot{\mathbf{X}}_\omega^{(i)} - \mathbf{D} \cdot \dot{\mathbf{A}} \right] - L(t) \\ &= \int d^3r \left[ \frac{(\mathbf{D} - \mathbf{P}^T)^2}{2\varepsilon_0} + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} - (\nabla \times \mathbf{A}) \cdot \mathbf{M} \right] \\ &+ \int d^3r \left[ \sum_{i=1}^2 \left[ \frac{1}{2} (\mathbf{Q}_\omega^{(i)})^2 + \frac{1}{2} \omega^2 (\mathbf{X}^{(i)})^2 \right] \right] \\ &+ \frac{1}{4\pi\varepsilon_0} \int d^3r \int d^3r' \frac{\nabla \cdot \mathbf{P}(\mathbf{r}, t) \nabla' \cdot \mathbf{P}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} \end{aligned} \quad (22)$$

If one write the Heisenberg equation for the vector potential  $\mathbf{A}$  the Maxwell equation (6) can be reobtained in the Heisenberg picture. Since it has been assumed that the volume contained the electromagnetic field and the medium is the unbounded space, we can expand the field operators of the system in terms of the plane waves. In accordance to the Coulomb gauge, the expansions of the vector potential and the displacement fields are

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\lambda=1}^2 \int d^3k \sqrt{\frac{\hbar}{2(2\pi)^3 \omega_{\mathbf{k}}}} [a_{\mathbf{k}\lambda}(t) e^{i\mathbf{k} \cdot \mathbf{r}} + a_{\mathbf{k}\lambda}^\dagger(t) e^{-i\mathbf{k} \cdot \mathbf{r}}] \mathbf{e}_{\mathbf{k}\lambda} \quad (23)$$

$$\mathbf{D}(\mathbf{r}, t) = -i \sum_{\lambda=1}^2 \int d^3k \sqrt{\frac{\hbar \omega_{\mathbf{k}}}{2(2\pi)^3}} [a_{\mathbf{k}\lambda}^\dagger(t) e^{-i\mathbf{k} \cdot \mathbf{r}} - a_{\mathbf{k}\lambda}(t) e^{i\mathbf{k} \cdot \mathbf{r}}] \mathbf{e}_{\mathbf{k}\lambda} \quad (24)$$



where  $\omega_{\mathbf{k}} = c|\mathbf{k}|$  and  $\mathbf{e}_{\mathbf{k}\lambda}$   $\lambda = 1, 2$  together with  $\mathbf{e}_{\mathbf{k}3} = \frac{\mathbf{k}}{|\mathbf{k}|}$  constitute an orthonormal basis. Also the expansions of the field operators of the magneto-dielectric medium are as follows

$$\mathbf{X}_{\omega}^{(i)}(\mathbf{r}, t) = \sum_{\nu=1}^3 \int d^3k \sqrt{\frac{\hbar}{2(2\pi)^3\omega_{\mathbf{k}}}} [b_{\mathbf{k}\nu}^{(i)}(\omega, t) e^{i\mathbf{k}\cdot\mathbf{r}} + (b_{\mathbf{k}\nu}^{(i)})^{\dagger}(\omega, t) e^{-i\mathbf{k}\cdot\mathbf{r}}] \mathbf{e}_{\mathbf{k}\nu} \quad (25)$$

$$\mathbf{Q}_{\omega}^{(i)}(\mathbf{r}, t) = -i \sum_{\nu=1}^3 \int d^3k \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2(2\pi)^3}} [b_{\mathbf{k}\nu}^{(i)}(\omega, t) e^{i\mathbf{k}\cdot\mathbf{r}} - (b_{\mathbf{k}\nu}^{(i)})^{\dagger}(\omega, t) e^{-i\mathbf{k}\cdot\mathbf{r}}] \mathbf{e}_{\mathbf{k}\nu} \quad (26)$$

Regarding to the commutation relations (20) and (21), it is clear that the ladder operators of the system satisfy the commutation rules

$$[a_{\mathbf{k}\lambda}(t), a_{\mathbf{k}'\lambda'}^{\dagger}(t)] = \delta_{\lambda\lambda'} \delta(\mathbf{k} - \mathbf{k}') \quad (27)$$

$$[b_{\mathbf{k}\nu}^{(i)}(\omega, t), (b_{\mathbf{k}'\nu'}^{(j)})^{\dagger}(\omega', t)] = \delta_{ij} \delta_{\nu\nu'} \delta(\omega - \omega') \delta(\mathbf{k} - \mathbf{k}') \quad (28)$$

## 5 multilayer magneto-dielectric media

The coupled constitutive relations (10) and the Maxwell equations can be solved using the forward and backward Laplace transformation[19]. Let us denote the backward and forward Laplace transformation of a time dependent operator  $O(t)$ , respectively by  $O^b(s)$  and  $O^f(s)$ . The operator valued functions  $O^b(s)$  and  $O^f(s)$  are defined as

$$O^b(s) = \int_0^{\infty} O(t) e^{-st} dt \quad O^f(s) = \int_0^{\infty} O(-t) e^{-st} dt \quad (29)$$

For a bi-anisotropic medium it is useful to express the Laplace transformations of the polarization fields  $\mathbf{P}(t)$  and  $\mathbf{M}(t)$  in terms of the Laplace transformations of the electric and magnetic fields  $\mathbf{E}(t)$  and  $\mathbf{H}(t)$ . Using the constitutive relations (10) we can write

$$\mathbf{P}^{f,b}(\mathbf{r}, s) = (\mathbf{P}'_N)^{f,b}(\mathbf{r}, s) + \eta^{(1)}(\mathbf{r}, s) \mathbf{E}^{f,b}(\mathbf{r}, s) + \eta^{(2)}(\mathbf{r}, s) \mathbf{H}^{f,b}(\mathbf{r}, s) \quad (30)$$

$$\mathbf{M}^{f,b}(\mathbf{r}, s) = (\mathbf{M}'_N)^{f,b}(\mathbf{r}, s) + \eta^{(3)}(\mathbf{r}, s)\mathbf{E}^{f,b}(\mathbf{r}, s) + \eta^{(4)}(\mathbf{r}, s)\mathbf{H}^{f,b}(\mathbf{r}, s) \quad (31)$$

where the superscripts  $f$  and  $b$  in the left hand are applied correspondingly for superscripts  $f$  and  $b$  in the right hand of this equation, respectively and

$$\eta^{(1)}(\mathbf{r}, s) = \chi^{(1)}(\mathbf{r}, s) + \mu_0\chi^{(2)}(\mathbf{r}, s)[I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1}\chi^{(4)}(\mathbf{r}, s) \quad (32)$$

$$\eta^{(2)}(\mathbf{r}, s) = \mu_0\chi^{(2)}(\mathbf{r}, s)[I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1} \quad (33)$$

$$\eta^{(3)}(\mathbf{r}, s) = [I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1}\chi^{(4)}(\mathbf{r}, s) \quad (34)$$

$$\eta^{(4)}(\mathbf{r}, s) = \mu_0\chi^{(3)}(\mathbf{r}, s)[I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1} \quad (35)$$

$$(\mathbf{P}'_N)^{f,b} = \mathbf{P}_N^{f,b}(\mathbf{r}, s) + \mu_0\chi^{(2)}(\mathbf{r}, s)[I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1}\mathbf{M}_N^{f,b}(\mathbf{r}, s) \quad (36)$$

$$(\mathbf{M}'_N)^{f,b}(\mathbf{r}, s) = [I - \mu_0\chi^{(3)}(\mathbf{r}, s)]^{-1}\mathbf{M}_N^{f,b}(\mathbf{r}, s) \quad (37)$$

where  $I$  is the unit tensor. It should be noted that since the susceptibility tensors are identically zero for  $t \leq 0$ , their backward Laplace transformation vanishes. Therefore only the forward Laplace transformation of the tensors are appeared in (30)-(37) that we have denoted them without superscript. Now combination of the constitutive relations (30), (31) with the Laplace transformations of Faraday's law and the Maxwell equation (6) gives the following six coupled differential equations for the cartesian components of the fields  $\mathbf{E}$  and  $\mathbf{H}$

$$\begin{bmatrix} T^{f,b}(\mathbf{r}, s) & Y^{f,b}(\mathbf{r}, s) \\ Z^{f,b}(\mathbf{r}, s) & W^{f,b}(\mathbf{r}, s) \end{bmatrix} \begin{bmatrix} \mathbf{E}^{f,b}(\mathbf{r}, s) \\ \mathbf{H}^{f,b}(\mathbf{r}, s) \end{bmatrix} = J^{f,b}(\mathbf{r}, s) \quad (38)$$

where

$$J^{f,b}(\mathbf{r}, s) = \begin{bmatrix} \mp\mu_0s(\mathbf{M}')_N^{f,b}(\mathbf{r}, s) \pm \mathbf{B}(\mathbf{r}, 0) \\ \pm s(\mathbf{P}')_N^{f,b}(\mathbf{r}, s) \mp \mathbf{D}(\mathbf{r}, 0) \end{bmatrix} \quad (39)$$

and

$$\begin{aligned} T_{ij}^{f,b} &= \sum_{k=1}^3 \epsilon_{ikj} \partial_k \pm \mu_0 s \eta_{ij}^{(3)} & Y^{f,b} &= \pm \mu_0 s (I + \eta^{(4)}) \\ W_{ij}^{f,b} &= \sum_{k=1}^3 \epsilon_{ikj} \partial_k \mp s \eta_{ij}^{(2)} & Z^{f,b} &= \mp s (\epsilon_0 I + \eta^{(1)}) \end{aligned} \quad (40)$$

where  $\epsilon_{ikj}$  is three dimensional Levi-civita symbol and the upper (lower) sign is applied for the forward (backward) Laplace transformation. Hereafter, the upper (lower) sign in any equation is used for the forward (backward) Laplace transformation and the superscripts  $f$  and  $b$  in the left hand of any equation are applied correspondingly for the superscripts  $f$  and  $b$  in the right hand of that equation, respectively. For a multilayer medium, that is when the susceptibility tensors are independent of the coordinates  $x, y$  and are piecewisely constant with respect to the coordinate  $z$ , The symmetry of configuration enables us to convert the coupled partial differential equations (38) into a set of first order ordinary differential equations with respect to the coordinate  $z$ . This can be achieved by expressing each field operators of the total system in terms of two dimensional Fourier transform with respect to coordinates  $x$  and  $y$ . For example for the electric field this becomes

$$\mathbf{E}^{(f,b)}(\mathbf{r}, s) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d^2k \underline{\mathbf{E}}^{f,b}(\mathbf{k}^{\parallel}, s, z) e^{\pm i\mathbf{k}^{\parallel} \cdot \mathbf{r}^{\parallel}} \quad (41)$$

where  $\mathbf{k}^{\parallel} = k_x \mathbf{i} + k_y \mathbf{j}$  and  $\mathbf{r}^{\parallel} = x \mathbf{i} + y \mathbf{j}$ . Applying this transformation into Eqs.(38), These equations are reduced to

$$\begin{bmatrix} \underline{T}^{f,b} & \underline{Y}^{f,b} \\ \underline{Z}^{f,b} & \underline{W}^{f,b} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{E}}^{f,b} \\ \underline{\mathbf{H}}^{f,b} \end{bmatrix} = \underline{J}^{f,b}(\mathbf{k}^{\parallel}, z, s) \quad (42)$$

where now

$$\begin{aligned} \underline{T}_{12}^{f,b} &= -\frac{\partial}{\partial z} \pm \mu_0 s \eta_{12}^{(3)}(z, s) & \underline{T}_{21}^{f,b} &= \frac{\partial}{\partial z} \pm \mu_0 s \eta_{21}^{(3)}(z, s) \\ \underline{T}_{13}^{f,b} &= \pm i k_y \pm \mu_0 s \eta_{13}^{(3)}(z, s) & \underline{T}_{31}^{f,b} &= \mp i k_y \pm \mu_0 s \eta_{31}^{(3)}(z, s) \\ \underline{T}_{23}^{f,b} &= \mp i k_x \pm \mu_0 s \eta_{23}^{(3)}(z, s) & \underline{T}_{32}^{f,b} &= \pm i k_x \pm \mu_0 s \eta_{32}^{(3)}(z, s) \\ \underline{T}_{ii}^{f,b} &= \pm \mu_0 s \eta_{ii}^{(3)}(z, s) & i &= 1, 2, 3 \end{aligned} \quad (43)$$

and

$$\begin{aligned} \underline{W}_{12}^{f,b} &= -\frac{\partial}{\partial z} \mp s \eta_{12}^{(2)}(z, s) & \underline{W}_{21}^{f,b} &= \frac{\partial}{\partial z} \mp s \eta_{21}^{(2)}(z, s) \\ \underline{W}_{13}^{f,b} &= \pm i k_y \mp s \eta_{13}^{(2)}(z, s) & \underline{W}_{31}^{f,b} &= \mp i k_y \mp s \eta_{31}^{(2)}(z, s) \\ \underline{W}_{23}^{f,b} &= \mp i k_x \mp s \eta_{23}^{(2)}(z, s) & \underline{W}_{32}^{f,b} &= \pm i k_x \mp s \eta_{32}^{(2)}(z, s) \\ \underline{W}_{ii}^{f,b} &= \mp s \eta_{ii}^{(2)}(z, s) & i &= 1, 2, 3 \end{aligned} \quad (44)$$

and the definitions of  $\underline{Y}^{f,b}, \underline{Z}^{f,b}$  are the same as in (40). The form of the differential equations frame (42) and the definitions (43),(44) shows that the third and the sixth equation in (42) are algebraic equations. This can be used to eliminate the  $z$  components of the fields  $\underline{\mathbf{E}}^{f,b}$  and  $\underline{\mathbf{H}}^{f,b}$  in the first, the second, the fourth and the fifth equation of the frame (42). If we solve the third equation together with the sixth equation of (42) for  $\underline{E}_z^{f,b}$  and  $\underline{H}_z^{f,b}$ , we obtain

$$\begin{aligned}\underline{E}_z^{f,b} &= \alpha_1 \underline{E}_x^{f,b} + \beta_1 \underline{E}_y^{f,b} + \gamma_1 \underline{H}_x^{f,b} + \delta_1 \underline{H}_y^{f,b} \pm \frac{W_{33} \underline{J}_3^{f,b} - Y_{33} \underline{J}_6^{f,b}}{T_{33} W_{33} - Z_{33} Y_{33}} \\ \underline{H}_z^{f,b} &= \alpha_2 \underline{E}_x^{f,b} + \beta_2 \underline{E}_y^{f,b} + \gamma_2 \underline{H}_x^{f,b} + \delta_2 \underline{H}_y^{f,b} \pm \frac{T_{33} \underline{J}_6^{f,b} - Z_{33} \underline{J}_3^{f,b}}{T_{33} W_{33} - Z_{33} Y_{33}}\end{aligned}\quad (45)$$

where

$$\begin{aligned}\alpha_1 &= \frac{Y_{33} Z_{31} - W_{33} T_{31}}{T_{33} W_{33} - Z_{33} Y_{33}}, \quad \beta_1 = \frac{Y_{33} Z_{32} - W_{33} T_{32}}{T_{33} W_{33} - Z_{33} Y_{33}}, \\ \gamma_1 &= \frac{Y_{33} W_{31} - W_{33} Y_{31}}{T_{33} W_{33} - Z_{33} Y_{33}}, \quad \delta_1 = \frac{Y_{33} W_{32} - W_{33} Y_{32}}{T_{33} W_{33} - Z_{33} Y_{33}} \\ \alpha_2 &= \frac{Z_{33} T_{31} - T_{33} Z_{31}}{T_{33} W_{33} - Z_{33} Y_{33}}, \quad \beta_2 = \frac{Z_{33} T_{32} - T_{33} Z_{32}}{T_{33} W_{33} - Z_{33} Y_{33}} \\ \gamma_2 &= \frac{Z_{33} Y_{31} - T_{33} W_{31}}{T_{33} W_{33} - Z_{33} Y_{33}}, \quad \delta_2 = \frac{Z_{33} Y_{32} - T_{33} W_{32}}{T_{33} W_{33} - Z_{33} Y_{33}}\end{aligned}\quad (46)$$

and we have used the new notation

$$\begin{aligned}\underline{T}_{13} &= \underline{T}_{13}^f, \quad \underline{T}_{31} = \underline{T}_{31}^f, \quad \underline{T}_{23} = \underline{T}_{23}^f, \quad \underline{T}_{32} = \underline{T}_{32}^f \\ \underline{W}_{13} &= \underline{W}_{13}^f, \quad \underline{W}_{31} = \underline{W}_{31}^f, \quad \underline{W}_{23} = \underline{W}_{23}^f, \quad \underline{W}_{32} = \underline{W}_{32}^f \\ \underline{Z}_{ij} &= \underline{Z}_{ij}^f, \quad \underline{Y}_{ij} = \underline{Y}_{ij}^f \quad i, j = 1, 2, 3 \\ \underline{T}_{ii} &= \underline{T}_{ii}^f, \quad \underline{W}_{ii} = \underline{W}_{ii}^f \quad i = 1, 2, 3\end{aligned}\quad (47)$$

Now substituting  $\underline{E}_z^{f,b}$  and  $\underline{H}_z^{f,b}$  from (45) into the first, the second, the fourth and the fifth equation of the frame (42) give us a set of the coupled first order differential equations for the other components of the fields  $\underline{\mathbf{E}}^{f,b}$  and  $\underline{\mathbf{H}}^{f,b}$  as

$$\frac{\partial}{\partial z} \Lambda^{f,b}(\mathbf{k}^{\parallel}, z, s) \pm \Theta(\mathbf{k}^{\parallel}, z, s) \Lambda^{f,b}(\mathbf{k}^{\parallel}, z, s) = G^{f,b}(\mathbf{k}^{\parallel}, z, s) \quad (48)$$

where

$$\Lambda^{f,b}(\mathbf{k}^{\parallel}, z, s) = \begin{bmatrix} \underline{E}_x^{f,b}(\mathbf{k}^{\parallel}, z, s) \\ \underline{E}_y^{f,b}(\mathbf{k}^{\parallel}, z, s) \\ \underline{H}_x^{f,b}(\mathbf{k}^{\parallel}, z, s) \\ \underline{H}_y^{f,b}(\mathbf{k}^{\parallel}, z, s) \end{bmatrix} \quad (49)$$

and the  $\Theta$ ,  $G^{f,b}$  are  $4 \times 4$  matrix and  $4 \times 1$  matrix ,respectively, with scalar elements given by

$$\begin{aligned} \Theta_{11} &= \mu_0 s \eta_{21}^{(3)} + \underline{T}_{23} \alpha_1 + \underline{Y}_{23} \alpha_2, \quad \Theta_{12} = \underline{T}_{22} + \underline{T}_{23} \beta_1 + \underline{Y}_{23} \beta_2 \\ \Theta_{13} &= \underline{Y}_{21} + \underline{T}_{23} \gamma_1 + \underline{Y}_{23} \gamma_2, \quad \Theta_{14} = \underline{Y}_{22} + \underline{T}_{23} \delta_1 + \underline{Y}_{23} \delta_2 \\ \Theta_{21} &= -\underline{T}_{11} - \underline{T}_{13} \alpha_1 - \underline{Y}_{13} \alpha_2, \quad \Theta_{22} = -\mu_0 s \eta_{12}^{(3)} - \underline{T}_{13} \beta_1 - \underline{Y}_{13} \beta_2, \\ \Theta_{23} &= -\underline{Y}_{11} - \underline{T}_{13} \gamma_1 - \underline{Y}_{13} \gamma_2, \quad \Theta_{24} = -\underline{Y}_{12} - \underline{T}_{13} \delta_1 - \underline{Y}_{13} \delta_2 \\ \Theta_{31} &= \underline{Z}_{21} + \underline{Z}_{23} \alpha_1 + \underline{W}_{23} \alpha_2, \quad \Theta_{32} = \underline{Z}_{22} + \underline{Z}_{23} \beta_1 + \underline{W}_{23} \beta_2 \\ \Theta_{33} &= -s \eta_{21}^{(2)} + \underline{Z}_{23} \gamma_1 + \underline{W}_{23} \gamma_2, \quad \Theta_{34} = \underline{W}_{22} + \underline{Z}_{23} \delta_1 + \underline{W}_{23} \delta_2 \\ \Theta_{41} &= -\underline{Z}_{11} - \underline{Z}_{13} \alpha_1 - \underline{W}_{13} \alpha_2, \quad \Theta_{42} = -\underline{Z}_{12} - \underline{Z}_{13} \beta_1 - \underline{W}_{13} \beta_2 \\ \Theta_{43} &= -\underline{W}_{11} - \underline{Z}_{13} \gamma_1 - \underline{W}_{13} \gamma_2, \quad \Theta_{44} = -s \eta_{12}^{(2)} - \underline{Z}_{13} \delta_1 - \underline{W}_{13} \delta_2 \end{aligned} \quad (50)$$

$$\begin{aligned} G_1^{f,b} &= \underline{J}_2^{f,b} + \frac{\underline{Z}_{33} \underline{Y}_{23}^{f,b} - \underline{T}_{23} \underline{W}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_3^{f,b} + \frac{\underline{T}_{23} \underline{Y}_{33} - \underline{Y}_{23} \underline{T}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_6^{f,b} \\ G_2^{f,b} &= -\underline{J}_1^{f,b} + \frac{\underline{T}_{13} \underline{W}_{33} - \underline{Y}_{13} \underline{Z}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_3^{f,b} + \frac{\underline{T}_{33} \underline{Y}_{13} - \underline{Y}_{33} \underline{T}_{13}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_6^{f,b} \\ G_3^{f,b} &= \underline{J}_5^{f,b} + \frac{\underline{W}_{23} \underline{Z}_{33} - \underline{Z}_{23} \underline{W}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_3^{f,b} + \frac{\underline{Z}_{23} \underline{Y}_{33} - \underline{W}_{23} \underline{T}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_6^{f,b} \\ G_4^{f,b} &= -\underline{J}_4^{f,b} + \frac{\underline{W}_{33} \underline{Z}_{13} - \underline{Z}_{33} \underline{W}_{13}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_3^{f,b} + \frac{\underline{W}_{13} \underline{T}_{33} - \underline{Z}_{13} \underline{Y}_{33}}{\underline{T}_{33} \underline{W}_{33} - \underline{Z}_{33} \underline{Y}_{33}} \underline{J}_6^{f,b} \end{aligned} \quad (51)$$

where  $\underline{J}_i^{f,b}$ ,  $i = 1, 2, 3, 4$  are the two dimensional Fourier transforms of the operators  $J_i^{f,b}$ ,  $i = 1, 2, 3, 4$  and  $J_i^{f,b}$ ,  $i = 1, 2, 3, 4$  are defined by (39).

Since for a multilayer medium the susceptibility tensors are piecewisely constant, regarding to the definitions (43), (44),(46) and (50) the matrix  $\Theta$  in (48) is independent of  $z$  for each layer. Therefore Eq.(48) in a layer is a set

of first order differential equations with constant coefficients. In this case the solution of Eq.(48) in each layer is the sum of two parts

$$\Lambda^{f,b}(\mathbf{k}^{\parallel}, z, s) = \Lambda_g^{f,b}(\mathbf{k}^{\parallel}, z, s) + \Lambda_p^{f,b}(\mathbf{k}^{\parallel}, z, s) \quad (52)$$

The part  $\Lambda_p^{f,b}$  is a special solution for Eq.(48) that can be written as

$$\Lambda_p^{f,b}(\mathbf{k}^{\parallel}, s, z) = \frac{1}{2\pi} \int_D dz' \int_{-\infty}^{+\infty} dq \frac{e^{iq(z-z')}}{iqI \pm \Theta(\mathbf{k}^{\parallel}, s)} G^{f,b}(\mathbf{k}^{\parallel}, s, z') \quad (53)$$

where the integration domain  $D$  is the width of the layer. The part  $\Lambda_g^{f,b}$  in (52) is the general solution of the homogeneous equation corresponding to Eq.(48) and is as

$$\Lambda_g^{f,b}(\mathbf{k}^{\parallel}, z, s) = \sum_{j=1}^4 C_j^{f,b}(\mathbf{k}^{\parallel}, s) R_j(\mathbf{k}^{\parallel}, s) \exp[\mp \Omega_j(\mathbf{k}^{\parallel}, s)z] \quad (54)$$

where  $R_j$ ,  $\Omega_j, j = 1, 2, 3, 4$  are the eigenvectors and the eigenvalues of the matrix  $\Theta$ , respectively, that is

$$\Theta(\mathbf{k}^{\parallel}, s) R_j(\mathbf{k}^{\parallel}, s) = \Omega_j(\mathbf{k}^{\parallel}, s) R_j(\mathbf{k}^{\parallel}, s) \quad j = 1, 2, 3, 4 \quad (55)$$

In (54) the operators  $C_j^{f,b}$ ,  $j = 1, 2, 3, 4$  should be obtained using the boundary conditions on the components of  $\Lambda^{f,b}$ . Regarding the definition of  $\Lambda^{f,b}$  in (49) the components of  $\Lambda^{f,b}$  should be continuous on the boundaries between layers. Also the components of  $\Lambda^{f,b}$  should be exponentially decay at  $z \rightarrow \pm\infty$ .

Consequently using the obtained backward and forward Laplace transformation of the four dimensional operator field  $\Lambda$ , we can express the time-space dependence of  $\Lambda$  as

$$\begin{aligned} \Lambda(\mathbf{r}, t) = & \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \int_{-\infty}^{+\infty} d^2k [\Lambda^f(\mathbf{k}^{\parallel}, z, -i\omega + 0^+) e^{i\mathbf{k}^{\parallel} \cdot \mathbf{r}^{\parallel}} \\ & + \Lambda^b(\mathbf{k}^{\parallel}, i\omega + 0^+, z) e^{-i\mathbf{k}^{\parallel} \cdot \mathbf{r}^{\parallel}}] \end{aligned} \quad (56)$$

Finally substituting the components of the operator  $\Lambda$  in Eqs.(45) the z-components of the fields  $E_z$  and  $H_z$  can be computed.

## 6 A magnetodielectric slab

Consider the region  $0 < z < d$  to be filled by a homogeneous magnetodielectric medium with susceptibilities tensors  $\chi^{(i)}(t)$   $i = 1, 2, 3, 4$  and the regions  $z < 0$  and  $z > d$  are free space. That is for the regions  $z < 0$  and  $z > d$  the susceptibilities tensor  $\chi^{(i)}(t)$   $i = 1, 2, 3, 4$  are identically zero. Therefore according to the relations (32)-(35) the tensors  $\eta^{(i)}$   $i = 1, 2, 3, 4$  are zero for the regions  $z < 0$  and  $z > d$ . Using the relations (40), (43),(44),(46) and (47) for the free spaces we deduce

$$\begin{aligned} \alpha_1 &= \beta_1 = \gamma_2 = \delta_2 = 0 \\ \gamma_1 &= -\frac{ik_y}{s\varepsilon_0}, \quad \delta_1 = \frac{ik_x}{s\varepsilon_0}, \quad \alpha_2 = \frac{ik_y}{s\mu_0}, \quad \beta_2 = -\frac{ik_x}{s\mu_0} \end{aligned} \quad (57)$$

Now from the definition of the matrix  $\Theta$  in (50) and using (40),(43), (44),(47), (50)and (57) this matrix for the regions  $z < 0$  and  $z > d$  becomes as

$$\Theta^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) = \begin{bmatrix} 0 & 0 & -\frac{k_x k_y}{s\varepsilon_0} & \mu_0 s + \frac{k_x^2}{s\varepsilon_0} \\ 0 & 0 & -\mu_0 s - \frac{k_y^2}{s\varepsilon_0} & \frac{k_x k_y}{s\varepsilon_0} \\ \frac{k_x k_y}{s\mu_0} & -\varepsilon_0 s - \frac{k_x^2}{s\mu_0} & 0 & 0 \\ \varepsilon_0 s + \frac{k_y^2}{s\mu_0} & -\frac{k_x k_y}{s\mu_0} & 0 & 0 \end{bmatrix} \quad (58)$$

The eigenvalues of  $\Theta^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})$  are as

$$\begin{aligned} \Omega_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) &= \Omega_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) = -\sqrt{\mathbf{k}_x^2 + \mathbf{k}_y^2 + s^2 \varepsilon_0 \mu_0} \\ \Omega_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) &= \Omega_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) = \sqrt{\mathbf{k}_x^2 + \mathbf{k}_y^2 + s^2 \varepsilon_0 \mu_0} \end{aligned} \quad (59)$$

and corresponding eigenvectors are

$$\begin{aligned} R_1^{(0)} &= \begin{bmatrix} -\frac{k_x^2 + s^2 \varepsilon_0 \mu_0}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ -\frac{k_x k_y}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ 0 \\ 1 \end{bmatrix} & R_2^{(0)} &= \begin{bmatrix} \frac{k_x k_y}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ \frac{k_y^2 + s^2 \varepsilon_0 \mu_0}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ 1 \\ 0 \end{bmatrix} \\ R_3^{(0)} &= \begin{bmatrix} \frac{k_x^2 + s^2 \varepsilon_0 \mu_0}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ \frac{k_x k_y}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ 0 \\ 1 \end{bmatrix} & R_4^{(0)} &= \begin{bmatrix} -\frac{k_x k_y}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ -\frac{k_y^2 + s^2 \varepsilon_0 \mu_0}{s\varepsilon_0 \sqrt{k_x^2 + k_y^2 + s^2 \varepsilon_0 \mu_0}} \\ 1 \\ 0 \end{bmatrix} \end{aligned} \quad (60)$$

For free spaces the coupling tensors  $\mathbf{f}_i, \mathbf{g}_i, i = 1, 2$  are zero and from the definitions (12) , (36) and (37) we deduce  $\mathbf{P}_N = \mathbf{P}'_N = \mathbf{M}_N = \mathbf{M}'_N = \mathbf{0}$ . Therefore according to the relation (39) we have

$$\mathbf{J}^{f,b} = \begin{bmatrix} \pm \mathbf{B}(\mathbf{r}, \mathbf{0}) \\ \mp \mathbf{D}(\mathbf{r}, \mathbf{0}) \end{bmatrix} \quad (61)$$

Using the relations (40), (41), (43), (44), (47) and (51) it is easy to show that for free space the source terms  $G_i^{f,b}, i = 1, 2, 3, 4$  in (??) are reduced to

$$\begin{aligned} G_1^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \pm \underline{\mathbf{B}}_y^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \pm \frac{i\mathbf{k}_x}{s\varepsilon_0} \underline{\mathbf{D}}_z^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \\ G_2^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \mp \underline{\mathbf{B}}_x^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \pm \frac{i\mathbf{k}_y}{s\varepsilon_0} \underline{\mathbf{D}}_z^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \\ G_3^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \mp \underline{\mathbf{D}}_y^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \pm \frac{i\mathbf{k}_x}{s\mu_0} \underline{\mathbf{B}}_z^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \\ G_4^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \pm \underline{\mathbf{D}}_x^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \pm \frac{i\mathbf{k}_y}{s\mu_0} \underline{\mathbf{B}}_z^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) \end{aligned} \quad (62)$$

where

$$\begin{aligned} \underline{\mathbf{D}}^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) &= -i \sum_{\lambda=1}^2 \int_{-\infty}^{+\infty} d\mathbf{k}_z \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{4\pi}} \left( \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\mp\mathbf{k}\lambda} - \mathbf{a}_{\pm\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\pm\mathbf{k}\lambda} \right) \mathbf{e}^{\pm i\mathbf{k}_z \mathbf{z}} \\ \underline{\mathbf{B}}^{f,b}(\mathbf{k}^{\parallel}, \mathbf{z}) &= \sum_{\lambda=1}^2 \int_{-\infty}^{+\infty} d\mathbf{k}_z \sqrt{\frac{\hbar}{4\pi\omega_{\mathbf{k}}}} (\pm i\mathbf{k} \times \mathbf{e}_{\pm\mathbf{k}\lambda}) \mathbf{a}_{\pm\mathbf{k}\lambda}(\mathbf{0}) \\ &+ (\pm i\mathbf{k} \times \mathbf{e}_{\mp\mathbf{k}\lambda}) \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}^{\pm i\mathbf{k}_z \mathbf{z}} \end{aligned} \quad (63)$$

Now using (62) and (63) it can be shown that the special answer  $\Lambda_p^{f,b}$  in (52) for free space is as

$$(\Lambda_p^{(0)})^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) = \int d\mathbf{k}_z \frac{\mathbf{e}^{\pm i\mathbf{k}_z \mathbf{z}}}{\pm i\mathbf{k}_z \mathbf{I} \pm \boldsymbol{\Theta}^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})} \mathbf{Q}^{f,b}(\mathbf{k}^{\parallel}, \mathbf{k}_z, \mathbf{s}) \quad (64)$$



where  $Q^{f,b}$  is a matrix  $4 \times 1$  with components

$$\begin{aligned}
Q_1^{f,b}(\mathbf{k}^{\parallel}, \mathbf{k}_z, \mathbf{s}) &= \\
&\pm \sum_{\lambda=1}^2 \sqrt{\frac{\hbar}{4\pi\omega_{\mathbf{k}}}} (\pm i\mathbf{k} \times \mathbf{e}_{\pm\mathbf{k}\lambda})_{\mathbf{y}} \mathbf{a}_{\pm\mathbf{k}\lambda}(\mathbf{0}) + (\pm i\mathbf{k} \times \mathbf{e}_{\mp\mathbf{k}\lambda})_{\mathbf{y}} \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0})] \\
&\pm \frac{ik_x}{s\varepsilon_0} \left[ -i \sum_{\lambda=1}^2 \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{4\pi}} \left( a_{\mp\mathbf{k}\lambda}^{\dagger}(0) \mathbf{e}_{\mp\mathbf{k}\lambda z} - \mathbf{a}_{\pm\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\pm\mathbf{k}\lambda z} \right) \right] \\
Q_2^{f,b}(\mathbf{k}^{\parallel}, \mathbf{k}_z, \mathbf{s}) &= \\
&\mp \sum_{\lambda=1}^2 \sqrt{\frac{\hbar}{4\pi\omega_{\mathbf{k}}}} (\pm i\mathbf{k} \times \mathbf{e}_{\pm\mathbf{k}\lambda})_{\mathbf{x}} \mathbf{a}_{\pm\mathbf{k}\lambda}(\mathbf{0}) + (\pm i\mathbf{k} \times \mathbf{e}_{\mp\mathbf{k}\lambda})_{\mathbf{x}} \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0})] \\
&\pm \frac{ik_y}{s\varepsilon_0} \left[ -i \sum_{\lambda=1}^2 \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{4\pi}} \left( a_{\mp\mathbf{k}\lambda}^{\dagger}(0) \mathbf{e}_{\mp\mathbf{k}\lambda z} - \mathbf{a}_{\pm\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\pm\mathbf{k}\lambda z} \right) \right] \\
Q_3^{f,b}(\mathbf{k}^{\parallel}, \mathbf{k}_z, \mathbf{s}) &= \mp \left[ -i \sum_{\lambda=1}^2 \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{4\pi}} \left( \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\mp\mathbf{k}\lambda y} - \mathbf{a}_{\pm\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\pm\mathbf{k}\lambda y} \right) \right] \\
&\pm \frac{ik_x}{\mu_0 s} \left[ \sum_{\lambda=1}^2 \sqrt{\frac{\hbar}{4\pi\omega_{\mathbf{k}}}} (\pm i\mathbf{k} \times \mathbf{e}_{\pm\mathbf{k}\lambda})_{\mathbf{z}} \mathbf{a}_{\pm\mathbf{k}\lambda}(\mathbf{0}) + (\pm i\mathbf{k} \times \mathbf{e}_{\mp\mathbf{k}\lambda})_{\mathbf{z}} \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \right] \\
Q_4^{f,b}(\mathbf{k}^{\parallel}, \mathbf{k}_z, \mathbf{s}) &= \pm \left[ -i \sum_{\lambda=1}^2 \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{4\pi}} \left( \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\mp\mathbf{k}\lambda x} - \mathbf{a}_{\pm\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \mathbf{e}_{\pm\mathbf{k}\lambda x} \right) \right] \\
&\pm \frac{ik_y}{\mu_0 s} \left[ \sum_{\lambda=1}^2 \sqrt{\frac{\hbar}{4\pi\omega_{\mathbf{k}}}} (\pm i\mathbf{k} \times \mathbf{e}_{\pm\mathbf{k}\lambda})_{\mathbf{z}} \mathbf{a}_{\pm\mathbf{k}\lambda}(\mathbf{0}) + (\pm i\mathbf{k} \times \mathbf{e}_{\mp\mathbf{k}\lambda})_{\mathbf{z}} \mathbf{a}_{\mp\mathbf{k}\lambda}^{\dagger}(\mathbf{0}) \right]
\end{aligned} \tag{65}$$

According to (52) and (54) the solution of differential equation (48) for the regions  $z < 0$ ,  $0 < z < d$  and  $z > d$  are respectively as

$$\begin{aligned}
(\Lambda^{(0)})^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \sum_{j=1}^4 \mathbf{C}_j^{\prime f,b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\mp \Omega_j^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{z}} + (\Lambda_{\mathbf{p}}^{(0)})^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) \\
\Lambda^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \sum_{j=1}^4 \mathbf{C}_j^{\prime f,b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\mp \Omega_j(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{z}} + \Lambda_{\mathbf{p}}^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) \\
(\Lambda^{(0)})^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z}) &= \sum_{j=1}^4 \mathbf{C}_j^{\prime\prime f,b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\mp \Omega_j^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{z}} + (\Lambda_{\mathbf{p}}^{(0)})^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z})
\end{aligned} \tag{66}$$

where  $\Lambda_p^{f,b}$  is given by (53). Because  $\Lambda^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{z})$  should be tend to zero at  $z = \pm\infty$  from the eigenvalues (59) we deduce

$$\begin{aligned}
C_3^{\prime f} &= C_4^{\prime f} = C_1^{\prime b} = C_2^{\prime b} = 0 \\
C_3^{\prime\prime b} &= C_4^{\prime\prime b} = C_1^{\prime\prime f} = C_2^{\prime\prime f} = 0
\end{aligned} \tag{67}$$

The boundary condition at  $z = 0$  give us

$$\begin{aligned}
&C_1^{\prime f}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_2^{\prime f}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + (\Lambda_{\mathbf{p}}^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, 0) \\
&= \sum_{j=1}^4 C_j^f(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j(\mathbf{k}^{\parallel}, \mathbf{s}) + \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, 0) \\
&C_3^{\prime b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_4^{\prime b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + (\Lambda_{\mathbf{p}}^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, 0) \\
&= \sum_{j=1}^4 C_j^b(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j(\mathbf{k}^{\parallel}, \mathbf{s}) + \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, 0)
\end{aligned} \tag{68}$$

Also the boundary condition at  $z = d$  implies

$$\begin{aligned}
&C_3^{\prime\prime f}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) d} + C_4^{\prime\prime f}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) d} \\
&+ (\Lambda_p^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, d) = \sum_{j=1}^4 \mathbf{C}_j^f(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_j(\mathbf{k}^{\parallel}, \mathbf{s}) d} + \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, d) \\
&C_1^{\prime\prime b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) d} + C_2^{\prime\prime b}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) d} \\
&+ (\Lambda_p^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, d) = \sum_{j=1}^4 \mathbf{C}_j^b(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_j(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_j(\mathbf{k}^{\parallel}, \mathbf{s}) d} + \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, d)
\end{aligned} \tag{69}$$

Using the relations (68) we can obtain the coefficients  $C^{f,b}(\mathbf{k}^{\parallel}, \mathbf{s})$   $\mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$  in terms of  $C_1'^f, C_2'^f, C_3'^b$  and  $C_4'^b$  as

$$\begin{bmatrix} C_1^f \\ C_2^f \\ C_3^f \\ C_4^f \end{bmatrix} = C_1'^f \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_2'^f \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \\ + \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{0}) - \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{0})) \quad (70)$$

$$\begin{bmatrix} C_1^b \\ C_2^b \\ C_3^b \\ C_4^b \end{bmatrix} = C_3'^b \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_4'^b \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \\ + \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{0}) - \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{0})) \quad (71)$$

where  $\Gamma(\mathbf{k}^{\parallel}, \mathbf{s})$  is a  $4 \times 4$  matrix are given by eigenvectors  $R_i(\mathbf{k}^{\parallel}, \mathbf{s})$ ,  $\mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$  as the following

$$\Gamma(\mathbf{k}^{\parallel}, \mathbf{s}) = \begin{bmatrix} R_1(\mathbf{k}^{\parallel}, \mathbf{s}) & R_2(\mathbf{k}^{\parallel}, \mathbf{s}) & R_3(\mathbf{k}^{\parallel}, \mathbf{s}) & R_4(\mathbf{k}^{\parallel}, \mathbf{s}) \end{bmatrix} \quad (72)$$

Also using Eq. (69) it is clear that one can write the coefficients  $C_i^{f,b}$   $i = 1, 2, 3, 4$  in terms of  $C_3''^f, C_4''^f, C_1''^b$  and  $C_2''^b$  as

$$\begin{bmatrix} C_1^f \\ C_2^f \\ C_3^f \\ C_4^f \end{bmatrix} = C_3''^f \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{d}} \\ + C_4''^f \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{d}} \\ + \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{d}) - \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{d})) \quad (73)$$

$$\begin{bmatrix} C_1^b \\ C_2^b \\ C_3^b \\ C_4^b \end{bmatrix} = C_1''^b \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{d}} \\ + C_2''^b \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{d}} \\ + \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{d}) - \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, \mathbf{d})) \quad (74)$$

where  $\Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s})$  and  $\Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s})$  are  $4 \times 4$  matrices defined by

$$\begin{aligned}\Delta^{-1} &= \begin{bmatrix} R_1 e^{-\Omega_1 d} & R_2 e^{-\Omega_2 d} & R_3 e^{-\Omega_3 d} & R_4 e^{-\Omega_4 d} \end{bmatrix} \\ \Pi^{-1} &= \begin{bmatrix} R_1 e^{\Omega_1 d} & R_2 e^{\Omega_2 d} & R_3 e^{\Omega_3 d} & R_4 e^{\Omega_4 d} \end{bmatrix}\end{aligned}\quad (75)$$

from Eqs.(70) and (73) we deduce

$$\begin{aligned}& C_1'^f \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_2'^f \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \\ & + \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, 0) - \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, 0)) \\ & = C_3''^f \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})d} \\ & + C_4''^f \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{-\Omega_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})d} \\ & + \Delta^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^f(\mathbf{k}^{\parallel}, \mathbf{s}, d) - \Lambda_{\mathbf{p}}^f(\mathbf{k}^{\parallel}, \mathbf{s}, d))\end{aligned}\quad (76)$$

where relate the coefficients  $C_1'^f, C_2'^f$  in region  $z < 0$  to the coefficients  $C_3''^f, C_4''^f$  in region  $z > d$ . Using Eqs.(71) and (74) we can write

$$\begin{aligned}& C_3'^b \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_3^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) + C_4'^b \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_4^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) \\ & + \Gamma^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, 0) - \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, 0)) \\ & = C_1''^b \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_1^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})d} \\ & + C_2''^b \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) \mathbf{R}_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s}) e^{\Omega_2^{(0)}(\mathbf{k}^{\parallel}, \mathbf{s})d} \\ & + \Pi^{-1}(\mathbf{k}^{\parallel}, \mathbf{s}) ((\Lambda_{\mathbf{p}}^{(0)})^b(\mathbf{k}^{\parallel}, \mathbf{s}, d) - \Lambda_{\mathbf{p}}^b(\mathbf{k}^{\parallel}, \mathbf{s}, d))\end{aligned}\quad (77)$$

where relate the coefficients  $C_3'^b, C_4'^b$  in region  $z < 0$  to the coefficients  $C_1''^b, C_2''^b$  in region  $z > d$ . Equation (76) is a frame of algebraic equations which one can find the coefficients  $C_1'^f, C_2'^f, C_3''^f, C_4''^f$  using this frame. Then, by insertion these coefficients in Eqs.(70) or (73) one can calculate the coefficients  $C_i^f$   $i = 1, 2, 3, 4$ . Also by solving the algebraic equations (77) the coefficients  $C_3'^b, C_4'^b, C_1''^b, C_2''^b$  can be fined and by instituting them in (71) or ((74)the coefficients  $C_i^b$   $i = 1, 2, 3, 4$  can be computed.

## 7 Summary and conclusion

A bi-anisotropic magnetodielectric medium was modeled by two independent reservoirs. Each reservoir contains a continuous set of three dimensional harmonic oscillators. In contrast of the damped polarization model, it is not

needed the electric and magnetic polarization fields of the medium to appear in the Lagrangian of the total system and the reservoirs solely constitute the degrees of freedom of the medium. The electric and magnetic polarization fields of the medium were obtained in terms of the dynamical variable modeling the medium and the coupling tensors of the medium and electromagnetic field. The constitutive relation of the bi-anisotropic magnetodielectric medium was obtained as a consequence of the Euler-Lagrange equations of the total system. For a multilayer medium, combination of the Maxwell equations and the constitutive relation of the medium was led to a frame of first order differential equations in terms of the electric and magnetic fields. By solving the differential equations in a standard way the electric and magnetic fields were computed for a bi-anisotropic magnetodielectric multilayer medium.

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